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Quiz for November 14, 2006

Find all of the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}.$$

ANSWER: To find the eigenvalues of A, we solve $\det(A - \lambda I) = 0$. We solve:

$$\det \begin{bmatrix} 1-\lambda & -1 \\ 1 & 3-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(3-\lambda) + 1 = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$(\lambda - 2)^2 = 0.$$

The only eigenvalue of A is $\boxed{\lambda = 2}$. To find the eigenvectors of A which belong to $\lambda = 2$ we find the null space of $A - 2I = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$. Apply the ERO $R2 \mapsto R2 + R1$ to obtain $\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}$. Mulitply row 1 by -1 to obtain $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$. The vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is an eigenvalue of A belonging to $\lambda = 2$ if and only if $x_1 = -x_2$. We conclude that the set of eigenvectors of A belonging to $\lambda = 2$ is

$$\boxed{\left\{ x_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} \middle| x_2 \in \mathbb{R} \right\}.}$$

We check that $A \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.