PRINT Your Name:____________________________________

Quiz for October 18, 2005

Express $v = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ as a linear combination of $u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$. (You are welcome to notice that u_1, u_2, u_3 form an orthogonal set of vectors.) Check your answer.

ANSWER: Suppose $v = c_1u_1 + c_2u_2 + c_3u_3$. Multiply both sides by u_1^{T} to see that $2 = 3c_1$; hence, $c_1 = \frac{2}{3}$, Multiply by u_2^{T} to see that $-1 = 2c_2$; hence $c_2 = \frac{-1}{2}$. Multiply by u_3^{T} to see that $1 = 6c_3$; hence $c_3 = \frac{1}{6}$. We check that

$$\frac{2}{3}u_1 - \frac{1}{2}u_2 + \frac{1}{6}u_3 = \frac{2}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 4+3-1 \\ 4+0+2 \\ 4-3-1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = v. \checkmark$$