PRINT Your Name:_____

**Quiz for October 11, 2005**

Let $A$ be a $5 \times 4$ matrix. What is the largest possible value for the rank of $A$? What is the smallest possible value for the nullity of $A$? Explain.

**ANSWER:** The rank of $A$ is the dimension of the columns space of $A$. The column space of $A$ is spanned by the four columns of $A$. Some subset of these four columns forms a basis for the column space of $A$; so the dimension of the column space of $A$ is less than or equal to $4$. We notice that this maximum value is attained sometimes, for example in

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The rank of $A$ plus the nullity of $A$ is equal to the number of columns of $A$. The nullity is minimized when the rank is maximized. So the minimum possible nullity for $A$ is $0$. Indeed, the indicated matrix has nullity zero.