Quiz 9, March 31, 2016

Let A and B be $n \times n$ matrices, with A non-singular. Does the column space of B have to equal the column space of AB? If the answer is yes, then give a complete, correct, proof. If the answer is no, then give an example.

Answer: No. Consider $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Observe that $AB = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$. We see that the column space of AB is the set of multiples of $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$; hence $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is in the column space of B; but is not in the column space of AB.