Quiz 2, January 26, 2015

Let A and B be 2×2 matrices. Prove or find a counterexample to the following statement:

$$(A-B)(A+B) = A^2 - B^2.$$

Answer. The statement is false. Numbers satisfy the given equation because numbers commute under multiplication. Any two matrices which do not commute under multiplication give a counterexample. So for example we take $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. We see

$$(A-B)(A+B) = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$A^2 - B^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

Thus, $(A-B)(A+B)$ is not always equal to $A^2 - B^2$, when A and B are 2×2 matrices.