

Math 544, Exam 1, Summer 2004

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet; start each problem on a **new sheet** of paper; and be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.

There are 5 problems. Each problem is worth 10 points. The exam is worth a total of 50 points. **SHOW** your work. **CIRCLE** your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail.**

I will leave your exam outside my office door by noon tomorrow, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the class is finished.

1. Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations. (Problems 1 and 2 have the same matrix A .)

$$A = \begin{bmatrix} 1 & 2 & 2 & 1 & 8 \\ 1 & 2 & 2 & 2 & 11 \\ 2 & 4 & 4 & 3 & 19 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}.$$

2. Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$A = \begin{bmatrix} 1 & 2 & 2 & 1 & 8 \\ 1 & 2 & 2 & 2 & 11 \\ 2 & 4 & 4 & 3 & 19 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 5 \\ 9 \end{bmatrix}.$$

3. Consider the system of linear equations.

$$\begin{aligned} 4x_1 + ax_2 &= 2 \\ ax_1 + x_2 &= -1. \end{aligned}$$

- (a) Which values for a cause the system to have no solution?
- (b) Which values for a cause the system to have exactly one solution?
- (c) Which values for a cause the system to have an infinite number of solutions?

Explain.

4. (True or False. If true, PROVE the result. If false, give a counter EXAMPLE.)
If A is a 2×2 matrix with $A^2 = 2A$, then $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ or $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.
5. (True or False. If true, PROVE the result. If false, give a counter EXAMPLE.)
If A and B are 2×2 matrices, then $A^2 - B^2 = (A - B)(A + B)$.