10. Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 1 \\ 3 & 6 & 6 & 2 \\ 1 & 2 & 1 & 3 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \text{and} \quad c = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}.$$

(a) (4 points) Find a basis for the null space of A.
(b) (3 points) What is the dimension of the null space of A?
(c) (4 points) Find a basis for the column space of A.
(d) (3 points) What is the dimension of the column space of A?
(e) (4 points) Find the general solution of $Ax = b$.
(f) (4 points) Find the general solution of $Ax = c$.

$$\left[\begin{array}{cccc|cc} 1 & 2 & 1 & 3 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 1 \\ 3 & 6 & 6 & 2 & 1 & 1 \\ 1 & 2 & 1 & 3 & 1 & 2 \end{array}\right]$$
$R2 \mapsto R2 - 2R1$
$R3 \mapsto R3 - 3R1$
$R4 \mapsto R4 - R1$

$$\left[\begin{array}{cccc|cc} 1 & 2 & 1 & 3 & 1 & 1 \\ 0 & 0 & 1 & -5 & -1 & -1 \\ 0 & 0 & 3 & -7 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$
$R1 \mapsto R1 - R2$
$R3 \mapsto R3 - 3R2$

$$\left[\begin{array}{cccc|cc} 1 & 2 & 0 & 8 & 2 & 2 \\ 0 & 0 & 1 & -5 & -1 & -1 \\ 0 & 0 & 0 & 8 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$
$R1 \to R1 - R3$
$R2 \mapsto R2 + \frac{5}{8}R3$
$R3 \mapsto \frac{1}{8}R3$

$$\left[\begin{array}{cccc|cc} 1 & 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & -\frac{3}{8} & -\frac{3}{8} \\ 0 & 0 & 1 & \frac{1}{8} & \frac{1}{8} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

(a) $x_1 = -2x_2$
$x_2 = x_2$
$x_3 = 0$
$x_4 = 0$

$$\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

(a)

(b) dim null space $A = 1$

(b)

(c) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 6 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ 3 \end{bmatrix} \right\}$

(d) dim col space $= 3$

(e) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -\frac{3}{8} \\ \frac{1}{8} \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

(f) no solution