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9. Let $v_1, \dots, v_m$ be vectors in $\mathbb{R}^n$. For each of the following questions, give one of the following answers: "definitely yes", "definitely no", or "sometimes". **Explain** your answer.

   (a) (3 points) Suppose $m = n$ and the vectors are linearly independent. Do the vectors span $\mathbb{R}^n$?

   Yes. The dimension theorem tells us that n linearly independent vectors in $\mathbb{R}^n$ are a basis for $\mathbb{R}^n$.

   (b) (3 points) Suppose $m = n + 1$. Are the vectors linearly independent?

   No. The "short fat" theorem tells us that $n+1$ vectors in $\mathbb{R}^n$ must be linearly dependent.

   (c) (3 points) Suppose $m = n + 1$. Do the vectors span $\mathbb{R}^n$?

   Sometimes $\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}2\\2\end{bmatrix}$ span $\mathbb{R}^2$

   $\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix}\begin{bmatrix}3\\0\end{bmatrix}$ do not span $\mathbb{R}^2$

   (d) (3 points) Suppose $m = n - 1$ and the vectors are linearly independent. Do the vectors span $\mathbb{R}^n$?

   No. If the vectors spanned $\mathbb{R}^n$ then they would be a basis for $\mathbb{R}^n$. But the dimension theorem tells us that every basis for $\mathbb{R}^n$ has n vectors.

   (e) (3 points) Suppose $m = n - 1$. Are the vectors linearly independent?

   Sometimes $\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}$ are linearly independent

   $\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}2\\0\\0\end{bmatrix}$ are not linearly independent.

   (f) (3 points) Suppose $m = n - 1$. Do the vectors span $\mathbb{R}^n$?

   No. Every spanning set can be reduced to a basis. But every basis for $\mathbb{R}^n$ has n vectors.