14. (5 points) Let $W = \{f\colon \mathbb{R} \to \mathbb{R} \mid f \text{ is differentiable}\}$. Is W a vector space? Explain.

Yes

The sum of 2 differentiable functions is differentiable.
A scalar times a differentiable function is differentiable.
The function $f(x)=0$ for all x is differentiable.

15. (5 points) Give an example of three 2×2 matrices A, B, and C, with A not the zero matrix, and $B \neq C$, but $BA = CA$.

$$\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -3 \\ 8 & -4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Take $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix}$ $C = \begin{bmatrix} 6 & -3 \\ 8 & -4 \end{bmatrix}$

we see $A \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $B \neq C$ but $BA = CA$.

16. (5 points) Let A and B be 2×2 matrices with A invertible. Does the column space of BA have to equal the column space of B? If the answer is yes, prove it. If the answer is no, give a counterexample.

Yes

Col space $BA \subseteq$ Col space B:

Every vector in col space BA has the form BAx for some x.
But $BAx = B(Ax)$ which is in the col space of B.

Col space $B \subseteq$ col space BA:

Every vector in the col space of B has the form Bx for some x. But $Bx = BA(A^{-1}x)$ which is in the col space of BA.