4. (5 points) Define "linear transformation". Use complete sentences.

The function $T$ from the vector space $V$ to the vector space $W$ is a linear transformation if 1) $T(v_1+v_2) = T(v_1)+T(v_2)$
2) $T(cv_1) = cT(v_1)$

for all $v_1, v_2 \in V$ and $c \in \mathbb{R}$.

5. (5 points) The *trace* of the square matrix $A$ is the sum of the numbers on its main diagonal. Let $V$ be the set of all $3 \times 3$ matrices with trace $0$. The set $V$ is a vector space. You do NOT have to prove this. Give a basis for $V$. NO justification is needed.

I will list 8 linearly independent elements of $V$. $V$ is a proper subspace of the set of all $3 \times 3$ matrices, so I will have listed the entire basis for $V$

$M_1 = \begin{bmatrix} 1&0&0\\0&-1&0\\0&0&0 \end{bmatrix}$, $M_2 = \begin{bmatrix} 1&0&0\\0&0&0\\0&0&-1 \end{bmatrix}$, $M_3 = \begin{bmatrix} 0&1&0\\0&0&0\\0&0&0 \end{bmatrix}$, $M_4 = \begin{bmatrix} 0&0&1\\0&0&0\\0&0&0 \end{bmatrix}$, $M_5 = \begin{bmatrix} 0&0&0\\1&0&0\\0&0&0 \end{bmatrix}$,

$M_6 = \begin{bmatrix} 0&0&0\\0&0&1\\0&0&0 \end{bmatrix}$, $M_7 = \begin{bmatrix} 0&0&0\\0&0&0\\1&0&0 \end{bmatrix}$, $M_8 = \begin{bmatrix} 0&0&0\\0&0&0\\0&1&0 \end{bmatrix}$

6. (5 points) Give an example of a matrix which is not diagonalizable. Explain why the matrix is not diagonalizable.

$A = \begin{bmatrix} 0&0\\1&0 \end{bmatrix}$ is not diagonalizable. The only evalue of $A$ is $\lambda = 0$

The e-space for $A$ belonging to $\lambda = 0$ is spanned by $\begin{bmatrix} 0\\1 \end{bmatrix}$

If $A$ were diagonalizable it would have to have two linearly independent evectors. But it doesn't.