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16. (10 points) Consider the linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ with

$$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}.$$

Find a matrix A with $T(v) = Av$ for all $v \in \mathbb{R}^2$.

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = 2T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) - T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) - T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ 0 \end{bmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 6 & -4 \\ 3 & 0 \end{pmatrix}$$