57

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Let

$$A = \begin{bmatrix} \frac{5}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{5}{2} \end{bmatrix}.$$

10. Find an invertible matrix S and a diagonal matrix D with $S^{-1}AS = D$.

11. Find a matrix B with $B^2 = A$.

$$\det(A-\lambda I) = \det\begin{pmatrix} \frac{5}{2}-\lambda & \frac{3}{2} \\ \frac{3}{2} & \frac{5}{2}-\lambda \end{pmatrix} = \left(\frac{5}{2}-\lambda\right)^2 - \frac{9}{4} = \frac{25}{4} - \frac{9}{4} - 5\lambda + \lambda^2$$

$$= \lambda^2 - 5\lambda + 4 = (\lambda-4)(\lambda-1) \quad \lambda = 1, 4$$

$$A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{8}{2} \\ \frac{8}{2} \end{bmatrix} = 4\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Take $S = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ $D = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$ 10

11 $A = SDS^{-1}$ Take $B = S\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}S^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$B = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$