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9. Find a basis for the vector space spanned by

$$v_1 = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \quad v_2 = \begin{bmatrix}0\\1\\0\\1\end{bmatrix}, \quad v_3 = \begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \quad \text{and} \quad v_4 = \begin{bmatrix}3\\2\\8\\-3\end{bmatrix}.$$

Show your work. Check your answer.

We need a basis for the column space of

$$\begin{bmatrix}1&0&1&3\\0&1&0&2\\1&0&0&8\\0&1&1&-3\end{bmatrix} \quad \begin{bmatrix}1&0&1&3\\0&1&0&2\\0&0&-1&5\\0&1&1&-3\end{bmatrix} \quad \begin{bmatrix}1&0&1&3\\0&1&0&2\\0&0&1&-5\\0&0&1&-5\end{bmatrix}$$

$R3 \mapsto R3-R1$ $\quad$ $R3 \mapsto -R3$, $R4 \mapsto R4-R2$ $\quad$ $R4 \mapsto R4-R3$, $R1 \mapsto R1-R3$

$$\begin{bmatrix}\underline{1}&0&0&8\\0&\underline{1}&0&2\\0&0&\underline{1}&-5\\0&0&0&0\end{bmatrix}$$

The null space is all $x = x_4 \begin{bmatrix}-8\\-2\\5\\1\end{bmatrix}$

Cols 1,2,3 have pivots so $v_1, v_2, v_3$ are a basis for the vector space spanned by $v_1, v_2, v_3$ and $v_4$

The fact that $\begin{bmatrix}-8\\-2\\5\\1\end{bmatrix}$ is in the null space tells us that

$v_4 = 8v_1 + 2v_2 - 5v_3$. This is true $8\begin{bmatrix}1\\0\\1\\0\end{bmatrix} + 2\begin{bmatrix}0\\1\\0\\1\end{bmatrix} - 5\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \begin{bmatrix}8-5\\2\\8\\2-5\end{bmatrix} =$

$= \begin{bmatrix}+3\\2\\8\\-3\end{bmatrix} = v_4$

This equation explains why $v_4$ is not part of the basis.