

9. Find bases for the null space, the column space, and the row space of

$$A = \begin{bmatrix} 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 4 & 6 \end{bmatrix}.$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = 0$$

$$x_2 = x_2$$

$$x_3 = -2x_4 - 3x_5$$

$$x_4 = x_4$$

$$x_5 = x_5$$

A basis for the null space is  $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

A basis for the row space is  $\{ [1 \ 0 \ 0 \ 0 \ 0], [0 \ 0 \ 1 \ 2 \ 3] \}$

A basis for the col. space is  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$