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8. Is the function $F\colon \mathbb{R}^2 \to \mathbb{R}^2$, which is defined by

$$F\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1^2 \\ x_1 x_2 \end{bmatrix},$$

a linear transformation? If so, explain why. If not, give an example to show that one of the rules of linear transformation fails to hold.

No

$$F\left(2\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = F\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$2F\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

different.

9. Is the function $F\colon \mathbb{R}^2 \to \mathbb{R}^3$, which is defined by

$$F\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix},$$

a linear transformation? If so, explain why. If not, give an example to show that one of the rules of linear transformation fails to hold.

Yes

$$F\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\right) = F\begin{bmatrix} x_1+y_1 \\ x_2+y_2 \end{bmatrix} = \begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ 0 \end{bmatrix}$$

the same

$$F\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + F\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ 0 \end{bmatrix}$$

$$F\left(c\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = F\begin{bmatrix} cx_1 \\ cx_2 \end{bmatrix} = \begin{bmatrix} cx_1 \\ cx_2 \\ 0 \end{bmatrix}$$

the same

$$cF\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = c\begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} cx_1 \\ cx_2 \\ 0 \end{bmatrix}$$