57 43

3. Define "linear transformation". Use complete sentences.

A linear transformation is a function T from a vector space V to a vector space W which satisfies

a) $T(v_1+v_2) = T(v_1)+T(v_2)$

b) $T(cv_1) = cT(v_1)$

for all $v_1, v_2 \in V$ and $c \in \mathbb{R}$.

4. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 nonsingular matrices, then $A+B$ is a nonsingular matrix.

False

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ are non singular, but $A+B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is sing.

5. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If U and V are subspaces of $\mathbb{R}^n$, then the intersection of U and V is also a subspace of $\mathbb{R}^n$.

True

a) U and V are subspaces so $0 \in U$ and $0 \in V$. Thus $0 \in U \cap V$

b) closure under +: If x and y are in $U \cap V$, then x and y are both in the vector space U, so $x+y \in U$. Also, x and y are both in the vector space V; so $x+y \in V$. Thus $x+y \in U \cap V$

c) closure under scalar mult.: If $x \in U \cap V$ and $c \in \mathbb{R}$, then x is in the vector space U so $cx \in U$. Also x is in the vector space V so $cx \in V$. Thus $cx \in U \cap V$.