

PRINT Your Name: _____

There are 4 pages. The problems are numbered from 1 to 8. The exam is worth 100 points. SHOW your work. **CIRCLE** your answer. **CHECK** your answer whenever possible.

1. Let

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 & 4 & 0 & 5 & 0 \\ 1 & 0 & 2 & 3 & 4 & 0 & 11 & 0 \\ 1 & 0 & 2 & 3 & 4 & 0 & 11 & 1 \end{bmatrix}$$

- (a) (10 points) Find a basis for the null space of A .
 (b) (10 points) Find a basis for the column space of A .

$R_2 \rightarrow R_2 - R_1$
 $R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 0 & 2 & 3 & 4 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 1 & 0 \end{bmatrix}$$

$R_2 \rightarrow \frac{1}{6}R_2$

$$\begin{bmatrix} 1 & 0 & 2 & 3 & 4 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 1 & 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 5R_2$
 $R_3 \rightarrow R_3 - 6R_2$

$$\begin{bmatrix} 1 & 0 & 2 & 3 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$x_1 = -2x_3 - 3x_4 - 4x_5$$

$$x_2 = x_2$$

$$x_3 = x_3$$

$$x_4 = x_4$$

$$x_5 = x_5$$

$$x_6 = x_6$$

$$x_7 = 0$$

$$x_8 = 0$$

A basis for the null space is

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

2 missing A

A basis for the col. sp. is

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 11 \\ 11 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

because leading ones occur in cols 1, 7, 8 in the reduced matrix