

10. Find the general solution of the following system of linear equations:

$$\begin{aligned} x_1 + x_2 - x_5 &= 1 \\ x_1 + 2x_2 + 2x_3 + x_4 + 2x_5 &= 2 \\ x_1 - x_3 + x_4 + x_5 &= 0. \end{aligned}$$

Also find three particular solutions of this system of equations. Be sure to check that all three of your particular solutions really satisfy the original system of linear equations.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & -1 & | & 1 \\ 1 & 2 & 2 & 1 & 2 & | & 2 \\ 1 & 0 & -1 & 1 & 1 & | & 0 \end{bmatrix} \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & -1 & | & 1 \\ 0 & 1 & 2 & 1 & 3 & | & 1 \\ 0 & -1 & -1 & 1 & 2 & | & -1 \end{bmatrix} \begin{array}{l} R_1 \rightarrow R_1 - R_2 \\ R_3 \rightarrow R_3 + R_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -2 & -1 & -4 & | & 0 \\ 0 & 1 & 2 & 1 & 3 & | & 1 \\ 0 & 0 & 1 & 2 & 5 & | & 0 \end{bmatrix}$$

$$\begin{array}{l} R_2 \rightarrow R_2 - 2R_3 \\ R_1 \rightarrow R_1 + 2R_3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 3 & 6 & | & 0 \\ 0 & 1 & 0 & -3 & -7 & | & 1 \\ 0 & 0 & 1 & 2 & 5 & | & 0 \end{bmatrix}$$

General solution

$$\begin{aligned} x_1 &= -3x_4 - 6x_5 \\ x_2 &= 3x_4 + 7x_5 + 1 \\ x_3 &= -2x_4 - 5x_5 \\ x_4 &= x_4 \\ x_5 &= x_5 \end{aligned}$$

Take  $x_4 = x_5 = 0$

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 1 \\ x_3 &= 0 \\ x_4 &= 0 \\ x_5 &= 0 \end{aligned}$$

Take  $x_4 = 1, x_5 = 0$

$$\begin{aligned} x_1 &= -3 \\ x_2 &= 4 \\ x_3 &= -2 \\ x_4 &= 1 \\ x_5 &= 0 \end{aligned}$$

Take  $x_4 = 0, x_5 = 1$

$$\begin{aligned} x_1 &= -6 \\ x_2 &= 8 \\ x_3 &= -5 \\ x_4 &= 0 \\ x_5 &= 1 \end{aligned}$$

Particular solutions

check

$$\begin{array}{lll} 0 + 1 - 0 = 1 \checkmark & -3 + 1 = -2 \checkmark & -6 + 8 - 1 = 1 \checkmark \\ 2 = 2 \checkmark & -3 + 8 - 4 + 1 = 2 \checkmark & -6 + 0 + 0 + 2 = -4 \checkmark \\ 0 = 0 \checkmark & -3 + 2 + 1 = 0 \checkmark & -6 + 5 + 1 = 0 \checkmark \end{array}$$