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4. Consider the function $T\colon \mathbb{R}^2 \to \mathbb{R}^3$, which is given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - 3x_2 \\ x_1 + 4 \\ 5x_2 \end{bmatrix}$.

   Is T a linear transformation? If so, then give a matrix A with $T(v) = Av$ for all $v \in \mathbb{R}^2$. If not, then give an example to show that one of the rules of linear transformation fails to hold.

T is not a linear trassformation because $T\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$

So $T\left(2\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = T\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$

and $2T\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = 2\begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 0 \end{bmatrix}$. We see $T\left(2\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) \neq 2\,T\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)$.

5. Consider the function $T\colon \mathbb{R}^2 \to \mathbb{R}^3$, which is given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - 3x_2 \\ x_1 + 4x_2 \\ 5x_2 \end{bmatrix}$.

   Is T a linear transformation? If so, then give a matrix A with $T(v) = Av$ for all $v \in \mathbb{R}^2$. If not, then give an example to show that one of the rules of linear transformation fails to hold.

T is a linear transformation because

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2 & -3 \\ 1 & 4 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

for all $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $x \mapsto Ax$ is a linear transformati for all matrices A.