6. True or False. (If true, give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^3$, then v_1, v_2, v_3 span $\mathbb{R}^3$.

True Let v be any vector in $\mathbb{R}^3$. The "short fat" theorem tells us that v_1, v_2, v_3, v is linearly dependent in $\mathbb{R}^3$. So there are numbers $c_1, c_2, c_3, c_4 \in \mathbb{R}$ with $c_1v_1 + c_2v_2 + c_3v_3 + c_4v = 0$, and at least one of the c's is not zero. If $c_4 = 0$, then $c_1v_1 + c_2v_2 + c_3v_3 = 0$ and this contradicts the hypothesis that v_1, v_2, v_3 are l.i.
So $c_4 \neq 0$ and $v = -\frac{c_1}{c}v_1 - \frac{c_2}{c}v_2 - \frac{c_3}{c}v_3$. Thus v is in the span of v_1, v_2, v_3.

7. True or False. (If true, give a proof. If false, give a counter example.) If A and B are 2×2 matrices, then $(AB)^{\mathrm{T}} = A^{\mathrm{T}}B^{\mathrm{T}}$.

False Take $A = [1\ 2]$ and $B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. We see

$(AB)^T = \left([1\ 2]\begin{bmatrix} 3 \\ 4 \end{bmatrix}\right)^T = [3+8]^T = [11]^T = [11]$

$A^TB^T = \begin{bmatrix} 1 \\ 2 \end{bmatrix}[3\ 4] = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$

not equal