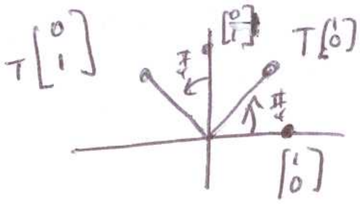
2. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which leaves the origin fixed and rotates the xy-plane by $\pi/4$ radians counterclockwise. What is the matrix M which has the property that $T(v) = Mv$ for all $v \in \mathbb{R}^2$?

$$M = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

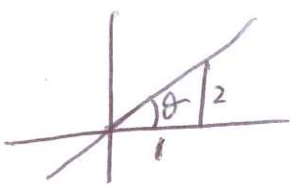
either because

$$M = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \quad \text{or}$$



3. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which reflects the xy-plane across the line $y = 2x$. What is the matrix M which has the property that $T(v) = Mv$ for all $v \in \mathbb{R}^2$?

$$M = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} = \begin{bmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$$



$\cos\theta = \frac{1}{\sqrt{5}}$ $\quad \sin\theta = \frac{2}{\sqrt{5}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{1}{5} - \frac{4}{5} = -\frac{3}{5}$

$\sin 2\theta = 2\sin\theta\cos\theta = \frac{4}{5}$

Extra check:

$M\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}$

$M\begin{bmatrix}-2\\1\end{bmatrix} = \begin{bmatrix}2\\-1\end{bmatrix}$