

7) No It is not closed under scalar multiplication.

$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  is in the set but  $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$  is not in the set.

8) Yes W is equal to the null space of  $\begin{bmatrix} a^T \\ b^T \end{bmatrix}$ .

The null space of a matrix is always a vector space.

9)

$\begin{bmatrix} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{bmatrix}$	$R_3 \rightarrow R_3 - 3R_1$ $R_2 \rightarrow \frac{1}{2}R_2$	$\begin{bmatrix} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & -7 & -3 & -3 & 0 & 1 \end{bmatrix}$	$R_1 \rightarrow R_1 - 4R_2$ $R_3 \rightarrow R_3 + 7R_2$	$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & -3 & \frac{7}{2} & 1 \end{bmatrix}$
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$R_2 \rightarrow R_2 - R_3$	$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 3 & -3 & -1 \\ 0 & 0 & \frac{1}{2} & -3 & \frac{7}{2} & 1 \end{bmatrix}$	$R_3 \rightarrow 2R_3$	$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 3 & -3 & -1 \\ 0 & 0 & 1 & -6 & 7 & 2 \end{bmatrix}$
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$A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ 3 & -3 & -1 \\ -6 & 7 & 2 \end{bmatrix}$

10) Suppose  $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$ . Then  $v_1^T (c_1 v_1 + c_2 v_2 + c_3 v_3) = 0$   
 Thus  $c_1 v_1^T v_1 + 0 + 0 = 0$ . But  $v_1^T v_1$  is a non-zero number so  $c_1 = 0$ .  
 Repeat this process  $v_2^T (c_1 v_1 + c_2 v_2 + c_3 v_3) = 0 \Rightarrow c_2 = 0$   
 $v_3^T (c_1 v_1 + c_2 v_2 + c_3 v_3) = 0 \Rightarrow c_3 = 0$ .

The only numbers  $c_1, c_2, c_3$  with  $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$  are  $c_1 = c_2 = c_3 = 0$ . We have proven that  $v_1, v_2, v_3$  are linearly independent.