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4. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If V and W are subspaces of $\mathbb{R}^n$, then the union $V \cup W$ is a subspace of $\mathbb{R}^n$.

False Let $V = \left\{ \begin{bmatrix} a \\ 0 \end{bmatrix} \middle| a \in \mathbb{R} \right\}$ and $W = \left\{ \begin{bmatrix} 0 \\ a \end{bmatrix} \middle| a \in \mathbb{R} \right\}$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \in V \cup W$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \in V \cup W$ but $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \notin V \cup W$

5. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If V and W are subspaces of $\mathbb{R}^n$, then the intersection $V \cap W$ is a subspace of $\mathbb{R}^n$.

True → 0 is in V and 0 is in W; hence 0 is in $V \cap W$.

→ Take x and y ∈ $V \cap W$. V is a vector space and x and y are in V, so x + y ∈ V. Similarly: W is a vector space and x and y are in W so x + y ∈ W. Thus, x + y ∈ $V \cap W$.

→ Take x ∈ $V \cap W$ and c ∈ $\mathbb{R}$.
V is a vector space so cx ∈ V. W is a vector space, so cx ∈ W
Thus cx ∈ $V \cap W$.

6. Let $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \middle| x^2 + y^2 = 1 \right\}$. Is V a subspace of $\mathbb{R}^2$? Justify your answer.

No $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \in V$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \in V$ but $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \notin V$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$