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8. Define "span". Use complete sentences.

The span of the vectors $v_1, \dots, v_p$ is the set of all linear combinations of $v_1, \dots, v_p$.

9. Define "linear combination". Use complete sentences.

The vector v is a linear combination of the vectors $v_1, \dots, v_p$ if there exist numbers $c_1, \dots, c_p$ with $v = c_1 v_1 + \dots + c_p v_p$.

10. Find h so that $v = \begin{bmatrix} 3 \\ -5 \\ h \end{bmatrix}$ is in the span of $v_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$.

I want h so that

$\left[\begin{array}{cc|c} 1 & -5 & 3 \\ 3 & -8 & -5 \\ -1 & 2 & h \end{array}\right]$ has a solution

$R3 \mapsto R3 + R2$
$R2 \mapsto R2 - 3R1$

$\left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 7 & -14 \\ 0 & -3 & 3+h \end{array}\right]$ $R2 \mapsto \frac{1}{7} R2$ $\left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & -3 & 3+h \end{array}\right]$

$R3 \mapsto R3 + 3R2$
$R1 \mapsto R1 + 5R2$

$\left[\begin{array}{cc|c} 1 & 0 & -7 \\ 0 & 1 & -2 \\ 0 & 0 & -3+h \end{array}\right]$

The system of equations has a solution only if $h = 3$.

If $h = 3$, then $-7v_1 - 2v_2 = -7\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} - 2\begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ 3 \end{bmatrix} = v$