Math 544, Exam 4, Summer 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 7 problems. Problem 1 is worth 8 points. Each of the other problems is worth 7 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your graded exam outside my office door. You may pick it up any time before the next class. **If you are interested, be sure to tell me.**

I will post the solutions on my website shortly after the class is finished.

1. Let $U \subseteq V$ be subspaces of $\mathbb{R}^n$ with $\dim U = \dim V$. Do U and V HAVE to be equal? If yes, prove your answer. If no, give an example.

2. Define "span". Use complete sentences. Include everything that is necessary, but nothing more.

3. Define "linear transformation". Use complete sentences. Include everything that is necessary, but nothing more.

4. Suppose $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation with
$$T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}2\\5\\8\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}3\\4\end{bmatrix}\right) = \begin{bmatrix}3\\-2\\1\end{bmatrix}.$$
Find $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$.

5. Find an orthogonal basis for the null space of $A = [1 \quad 1 \quad 1 \quad 2]$. **CHECK your answer.**

6. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be reflection across the line $y = \sqrt{3}\,x$. Find a matrix M with $T(v) = Mv$ for all vectors v in $\mathbb{R}^2$. **CHECK your answer.**

7. Let $A = \begin{bmatrix}-1 & -10\\5 & 14\end{bmatrix}$. Find a matrix B with $B^2 = A$. **CHECK your answer.**