Math 544, Exam 1, Summer 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 6 problems. Problem 1 is worth 10 points. Each of the other problems is worth 8 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your graded exam outside my office door. You may pick it up any time before the next class. **If you are interested, be sure to tell me.**

I will post the solutions on my website shortly after the class is finished.

1. Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$A = \begin{bmatrix} 1 & 4 & 5 & 1 & 1 & 5 \\ 1 & 4 & 5 & 2 & 1 & 8 \\ 1 & 4 & 5 & 2 & 2 & 8 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

2. Consider the system of linear equations.

$$\begin{aligned} x_1 + 4ax_2 &= 4 \\ ax_1 + \quad x_2 &= 2. \end{aligned}$$

   (a) Which values for a cause the system to have no solution?
   (b) Which values for a cause the system to have exactly one solution?
   (c) Which values for a cause the system to have an infinite number of solutions?

Explain thoroughly.

3. Are the vectors

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$$

   linearly independent? **Explain thoroughly.**

4. (True or False. If true, PROVE the result. If false, give a counter EXAMPLE.) If A is a 2×2 matrix with $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, then $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

5. Let A and B be symmetric $n \times n$ matrices. Suppose that AB is also a symmetric matric. Prove that $AB = BA$.

6. Let v_1, v_2, v_3, v_4 be vectors in $\mathbb{R}^5$. Suppose that v_1, v_2, v_3 are linearly dependent. Do the vectors v_1, v_2, v_3, v_4 HAVE to be linearly dependent? If yes, PROVE the result. If no, show an EXAMPLE.