Exam 4, Summer 2003, Math 544

PRINT Your Name:______________________________

Please also write your name on the back of the exam.

There are 9 problems on 5 pages. Problem 2 is worth 10 points. Each of the other problems is worth 5 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office door later today (surely by 5:00 PM), you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the class is finished.

1. Define "basis". Use complete sentences.

2. State any TWO of the four theorems about dimension. Use complete sentences.

3. **Yes or No.** Let v_1, v_2, v_3 be linearly independent vectors in $\mathbb{R}^n$ and let $T\colon \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Is it always true that $T(v_1)$, $T(v_2)$, $T(v_3)$ are linearly independent vectors? If yes, prove it. If no, give an example.

4. **Yes or No.** Let v_1, v_2, v_3 be vectors in $\mathbb{R}^n$ and let $T\colon \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Suppose that $T(v_1)$, $T(v_2)$, $T(v_3)$ are linearly independent vectors in $\mathbb{R}^m$. Do the vectors v_1, v_2, v_3 have to be linearly independent? If yes, prove it. If no, give an example.

5. **Yes or No.** Let A be a 3×4 matrix. Suppose that the column space of A has dimension 2. Does the system of equations $Ax = b$ have at least one solution for every vector b in $\mathbb{R}^3$? Explain.

6. **Yes or No.** Let A and B be 2×2 matrices. Does

$$\text{the null space of } A \subseteq \text{the null space of } AB$$

always happen? If yes, prove it. If no, give an example.

7. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix}$. Find a matrix B with BA equal to the identity matrix. You may do the problem any way you like, but you might want to notice that the columns of A are an orthogonal set.

8. Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$.

9. Find an orthogonal basis for the null space of $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 2 & 4 \end{bmatrix}$.