Exam 3,   Summer 2003,   Math 544

PRINT Your Name:_____

Please also write your name on the back of the exam.

There are 9 problems on 5 pages. Problem 4 is worth 10 points. Each of the other problems is worth 5 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office door later today (surely by 5:00 PM), you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the class is finished.

1. Define "linearly independent". Use complete sentences.

2. Define "null space". Use complete sentences.

3. Define "span". Use complete sentences.

4. Let $A = \begin{bmatrix} 1 & 2 & 3 & 3 & 6 & 7 \\ 1 & 2 & 3 & 3 & 6 & 8 \\ 2 & 4 & 6 & 6 & 12 & 15 \\ 1 & 2 & 3 & 4 & 11 & 1 \end{bmatrix}$. Find a basis for the null space of A. Find a basis for the column space of A. Find a basis for the row space of A. Express each column of A as a linear combination of the basis you have chosen for the column space of A. Express each row of A as a linear combination of the basis you have chosen for the row space of A.

5. Let A and B be 2×2 matrices. Does

    the column space of $AB \subseteq$ the column space of A

    always happen? If yes, prove it. If no, give an example.

6. Let A and B be 2×2 matrices. Does

the null space of $AB \subseteq$ the null space of A

always happen? If yes, prove it. If no, give an example.

7. True or False. Let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \ x_2 x_3 = 0 \right\}$. Is W a vector space? If yes, explain why. If no, give an example to show that the rules of vector space do not hold.

8. True or False. Let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \ x_1 + x_2 = x_3 \right\}$. Is W a vector space? If yes, explain why. If no, give an example to show that the rules of vector space do not hold.

9. Let A be a 2×3 matrix. Suppose that the column space of A has dimension 2 . Is the system of equations $Ax = b$ consistent for every choice of the vector b in $\mathbb{R}^2$? Explain.