Math 544, Summer 2003, Exam 2

PRINT Your Name:_____

Please also write your name on the back of the exam.

There are 10 problems on 5 pages. Each problem is worth 5 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office door later today, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the class is finished.

1. Define "linearly independent". Use complete sentences.

2. Define "non-singular". Use complete sentences.

3. Let A be an $n \times n$ matrix. List three conditions which are equivalent to the statement "A is non-singular". (I expect three new conditions **in addition** to "A is non-singular". Also, I do **not** expect you to repeat your answer to problem 2.)

4. Find the GENERAL solution of the following system of linear equations. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$\begin{array}{rrrrrcc} x_1 & +3x_2 & +4x_3 & +2x_4 & +4x_5 & = & 16 \\ x_1 & +3x_2 & +4x_3 & +3x_4 & +6x_5 & = & 21 \\ 2x_1 & +6x_2 & +8x_3 & +5x_4 & +10x_5 & = & 37 \end{array}$$

5. Are the vectors $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, $v_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$ linearly independent? Explain.

6. True or False. (If true, explain why or give a proof. If false, give a counter example.) If A, B are 2×2 invertible matrices, then AB is an invertible matrix.

7. True or False. (If true, explain why or give a proof. If false, give a counter example.) If the vectors v_1, v_2, and v_3 are linearly independent, then the vectors $v_1 + v_3$, $v_2 + v_3$, and $v_1 + v_2$ are also linearly independent.

8. True or False. (If true, explain why or give a proof. If false, give a counter example.) If the vectors v_1, v_2, and v_3 are linearly independent, then the vectors $v_1 - v_3$, $v_3 - v_2$, and $v_2 - v_1$ are also linearly independent.

9. True or False. (If true, explain why or give a proof. If false, give a counter example.) If A, B, and C are 2×2 matrices, with $A \neq 0$ and $BA = CA$, then $B = C$.

10. True or False. (If true, explain why or give a proof. If false, give a counter example.) If A, B are 2×2 matrices, with AB equal to the identity matrix, then BA is also equal to the identity matrix.