Math 544, Summer 2001, Exam 3

PRINT Your Name:________________________________

There are 10 problems on 4 pages. Each problem is worth 5 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators.

1. Define "one-to-one". Use complete sentences.

2. Define "onto". Use complete sentences.

3. Suppose A is an $n \times n$ matrix and $Ax = 0$ has a unique solution. Let b be a vector in $\mathbb{R}^n$. How many solutions does $Ax = b$ have? Explain.

4. Are
$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \text{and} \quad v_4 = \begin{bmatrix} 3 \\ 2 \\ 8 \\ -3 \end{bmatrix}$$
linearly dependent or linearly independent? Show your work. Check your answer.

5. Let $A = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 0 \\ 3 & 2 & 6 \end{bmatrix}$. Find A^{-1}. Check your answer.

6. Let $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_2 \\ 2x_1 + 5x_2 \end{bmatrix}$. Find a formula for T^{-1}. Check your answer.

7. The function $T \colon \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation with $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$. Find a matrix A with $T(v) = Av$ for all $v \in \mathbb{R}^2$. Check your answer.

8. Suppose $v_1, \ldots, v_n$ are linearly independent vectors in $\mathbb{R}^n$. Do $v_1, \ldots, v_n$ have to span $\mathbb{R}^n$? Explain.

9. Suppose $v_1, \ldots, v_n$ are vectors in $\mathbb{R}^m$, which span $\mathbb{R}^m$. Do $v_1, \ldots, v_n$ have to be linearly independent? Explain.

10. Let A, B, and C be 2×2 matrices with A not equal to the zero matrix. Is it possible for $AB = AC$, but $B \neq C$? If possible, find such matrices. If not possible, explain why not.