Math 544, Summer 2001, Exam 2

PRINT Your Name:__________________________________

There are 10 problems on 5 pages. Each problem is worth 5 points. SHOW your work. *CIRCLE* your answer. **CHECK** your answer whenever possible.

No Calculators.

1. Define "linear combination". Use complete sentences.

2. Define "linearly independent". Use complete sentences.

3. Define "linear transformation". Use complete sentences.

4. Consider the function $T\colon \mathbb{R}^2 \to \mathbb{R}^3$, which is given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - 3x_2 \\ x_1 + 4 \\ 5x_2 \end{bmatrix}.$$

Is T a linear transformation? If so, then give a matrix A with $T(v) = Av$ for all $v \in \mathbb{R}^2$. If not, then give an example to show that one of the rules of linear transformation fails to hold.

5. Consider the function $T\colon \mathbb{R}^2 \to \mathbb{R}^3$, which is given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - 3x_2 \\ x_1 + 4x_2 \\ 5x_2 \end{bmatrix}.$$

Is T a linear transformation? If so, then give a matrix A with $T(v) = Av$ for all $v \in \mathbb{R}^2$. If not, then give an example to show that one of the rules of linear transformation fails to hold.

6. The function $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation with $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$. Find a matrix A with $T(v) = Av$ for all $v \in \mathbb{R}^2$.

7. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.)
Let A , B , and C be 2×2 matrices with A not equal to the zero matrix. If $AB = AC$, then $B = C$.

8. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.)
If v_1, v_2, v_3, v_4 are in $\mathbb{R}^4$ and v_3 is *not* a linear combination of v_1, v_2, v_4 , then $\{v_1, v_2, v_3, v_4\}$ is linearly independent.

9. Are

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \text{and} \quad v_4 = \begin{bmatrix} 3 \\ 2 \\ 8 \\ -3 \end{bmatrix}$$

linearly dependent or linearly independent? Show your work. Check your answer.

10. Find the general solution of the following system of linear equations:

$$\begin{array}{llllll} x_1 & + x_2 & & & - x_5 & = 1 \\ x_1 & +2x_2 & +2x_3 & + x_4 & +2x_5 & = 2 \\ x_1 & & - x_3 & + x_4 & + x_5 & = 0. \end{array}$$

Also find **three** particular solutions of this system of equations. **Be sure to check** that all three of your particular solutions really satisfy the original system of linear equations.