Exam 4, Math 544, Spring, 2003

PRINT Your Name:________________________________

Please also write your name on the back of the exam.

There are 8 problems on 4 pages. Problem 1 is worth 15 points. Each of the other problems is worth 5 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, send me an e-mail.

I will leave your exam outside my office door about 6 PM SATURDAY, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the exam is finished.

1. Find a matrix B with $B^2 = A$ for $A = \begin{bmatrix} 13 & 18 \\ -6 & -8 \end{bmatrix}$. I expect you to write down the four entries of B. **Check your answer.**

2. Define "linear transformation". Use complete sentences.

3. Define "eigenvalue". Use complete sentences.

4. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be reflection across the line $y = -x$. Find a matrix A with $T(v) = Av$ for all $v \in \mathbb{R}^2$. **Check your answer.**

5. Prove that every 3×3 skew-symmetric matrix is singular. (Recall that the matrix A is *skew-symmetric* if $A^{\mathrm{T}} = -A$.)

6. Prove that every 2×2 symmetric matrix has at least one real eigenvalue. (Recall that the matrix A is *symmetric* if $A^{\mathrm{T}} = A$.)

7. Let v_1 , v_2 , and v_3 be an orthogonal set of non-zero vectors. Prove that v_1 , v_2 , and v_3 are linearly independent.

8. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.) If A and B are square matrices, then the null space of $A + B$ is contained in the intersection of the null space of A and the null space of B .