Math 544 Exam 2 Spring 2003

PRINT Your Name:________________________________

Please also write your name on the back of the exam.
There are 8 problems on 4 pages. Problem 4 is worth 15 points. Each of the other problems is worth 5 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**
If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, send me an e-mail.
I will leave your exam outside my office door about 6PM today, you may pick it up any time between then and the next class.
I will post the solutions on my website shortly after the exam is finished.

1. Define "linearly independent". Use complete sentences.

2. Define "non-singular". Use complete sentences.

3. Let A be an $n \times n$ matrix. List three conditions which are equivalent to the statement "A is non-singular". (I expect three new conditions **in addition** to "A is non-singular". Also, I do **not** expect you to repeat your answer to problem 2.)

4. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$.
   (a) Find the GENERAL solution of the system of equations $Ax = b$.
   (b) List two SPECIFIC solutions of $Ax = b$, if possible. CHECK that the specific solutions satisfy the equations.
   (c) Are the columns of A linearly independent? Explain.
   (d) List vectors $v_1, \ldots, v_r$ so that the null space of A is the span of $v_1, \ldots, v_r$. (You pick the appropriate number for r .)
   (e) Is b in the column space of A? Explain.

5. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.) If U and V are subspaces of $\mathbb{R}^2$, then the union $U \cup V$ is also a subspace of $\mathbb{R}^2$.

6. Let v_1, v_2, and v_3 be non-zero vectors in $\mathbb{R}^4$. Suppose that $v_i^{\mathrm{T}} v_j = 0$ for all subscripts i and j with $i \neq j$. Prove that v_1, v_2, and v_3 are linearly independent.

7. Let

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 \;\middle|\; \begin{array}{cccccc} 1x_1 & +2x_2 & +3x_3 & = & x_4 & \text{and} \\ 2x_1 & +3x_2 & +4x_3 & = & 2x_4 & \end{array} \right\}.$$

Is W a vector space? Explain.

8. Let

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; x_2 = x_1 x_3 \right\}.$$

Is W a vector space? Explain.