Math 544, Spring 2002, Final Exam

PRINT Your Name:______________________________

The exam is worth 100 points. There are 18 problems on 9 pages. SHOW your work. *CIRCLE* your answer. **CHECK** your answer whenever possible. **No Calculators.**

Your grade for the course will be available from VIP by the end of the week. Also, If you send me an e-mail (kustin@math.sc.edu) asking for your grade, I will e-mail it to you as soon as it is available.

1. (8 points) Let A be an $n \times n$ matrix. List 8 statements that are equivalent to the statement " A is invertible".

2. (5 points) Define "onto". Use complete sentences.

3. (5 points) Define "linearly independent". Use complete sentences.

4. (5 points) Define "linear transformation". Use complete sentences.

5. (5 points) The *trace* of the square matrix A is the sum of the numbers on its main diagonal. Let V be the set of all 3×3 matrices with trace 0. The set V is a vector space. You do NOT have to prove this. Give a basis for V. NO justification is needed.

6. (5 points) Give an example of a matrix which is not diagonalizable. Explain why the matrix is not diagonalizable.

7. (8 points) Let $A = \begin{bmatrix} 7 & 6 \\ -3 & -2 \end{bmatrix}$. Diagonalize A. Find a matrix B with $B^2 = A$. **Check your answer.**

8. (9 points) Let

$$A = \begin{bmatrix} 1 & 0 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 4 & 6 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}.$$

Solve $Ax = b$. Find a basis for the null space of A. Find a basis for the column space of A. Find a basis for the row space of A. **Check your answer.**

9. (5 points) Find an orthogonal set which is a basis for the null space of $A = [1 \quad -1 \quad 1 \quad -1]$. **Check your answer.**

10. (5 points) Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}.$$

Find the least squares solution for $Ax = b$. **Be sure to check your answer,** call it $\hat{x}$, by verifying that $b - A\hat{x}$ is orthogonal to the column space of A.

11. (5 points) Is it possible that all solutions of a homogeneous system of ten linear equations in twelve variables are multiples of one fixed non-zero solution? Explain.

12. (5 points) Give an example of a two by two matrix A, with no zero entries, such that A^2 is equal to the identity matrix.

13. (5 points) Let

$$A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

Solve $Ax = b$. (You may do the problem any way you wish; however, you may find it helpful to notice that the columns of A form an orthogonal set.) **Check your answer.**

14. (5 points) Let $W = \{f\colon \mathbb{R} \to \mathbb{R} \mid f \text{ is differentiable}\}$. Is W a vector space? Explain.

15. (5 points) Give an example of three 2×2 matrices A, B, and C, with A not the zero matrix, and $B \neq C$, but $BA = CA$.

16. (5 points) Let A and B be 2×2 matrices with A invertible. Does the column space of BA have to equal the column space of B? If the answer is yes, prove it. If the answer is no, give a counterexample.

17. (5 points) True or False. (If true, give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^4$ and $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ is a linear transformation, then $T(v_1), T(v_2), T(v_3)$ are linearly independent vectors in $\mathbb{R}^4$.

18. (5 points) True or False. (If true, give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly dependent vectors in $\mathbb{R}^4$ and $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ is a linear transformation, then $T(v_1), T(v_2), T(v_3)$ are linearly dependent vectors in $\mathbb{R}^4$.