Math 544, Spring 2002, Exam 3

PRINT Your Name:________________________________

There are 10 problems on 4 pages. Each problem is worth 5 points. SHOW your work. *CIRCLE* your answer. **CHECK** your answer whenever possible. **No Calculators.**

1. Define "basis". Use complete sentences.

2. Define "null space". Use complete sentences.

3. Let $W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; a_1 + a_2 = {a_3}^2 \right\}$. Is W a vector space? Explain.

4. Let $W = \{f\colon \mathbb{R} \to \mathbb{R} \mid f \text{ is continuous}\}$. Is W a vector space? Explain.

5. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the function $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ \sin y \end{bmatrix}$. Is T a linear transformation? Explain.

6. Give an example of three 2×2 matrices A, B, and C, with A not the zero matrix, and $B \neq C$, but $AB = AC$.

7. Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 9 \\ 3 & 6 & 6 \end{bmatrix}$. Find a basis for the null space of A. Find a basis for the column space of A.

8. Let V be the subspace of $\mathbb{R}^3$ which is spanned by

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 4 \\ 9 \\ 6 \end{bmatrix}, \quad \text{and} \quad v_4 = \begin{bmatrix} 6 \\ 13 \\ 12 \end{bmatrix}.$$

   Find a basis for V.

9. Let $v_1, \dots, v_n$ be n linearly independent vectors in $\mathbb{R}^n$. Prove that $v_1, \dots, v_n$ is a basis for $\mathbb{R}^n$.

10. Let A and B be 2×2 matrices with A invertible. Does the columns space of AB have to equal the column space of B? If the answer is yes, prove it. If the answer is no, give a counterexample.