

PRINT Your Name: \_\_\_\_\_

**Quiz for May 30, 2012**

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** Write in complete sentences. Express your work in a neat and coherent manner.

Let  $A$  and  $B$  be  $n \times n$  matrices show that  $AB$  is non-singular. **Prove** that  $A$  is non-singular.

**ANSWER:** We first show that  $B$  is non-singular. Suppose that  $v$  is a vector with  $Bv = 0$ . Multiply both sides of the equation on the left by  $A$  to get  $A(Bv) = A0$ . It follows that  $(AB)v = 0$ . The matrix  $AB$  is non-singular; hence,  $v = 0$ .

The matrix  $B$  is non-singular; hence, according to the non-singular matrix theorem,  $B$  has an inverse. Let  $C$  be  $B$ 's inverse.

Now we show that  $A$  is non-singular. Suppose  $v$  is a vector with  $Av = 0$ . We know that  $BC = I$ . We have  $0 = Av = AB(Cv)$ . The matrix  $AB$  is non-singular by hypothesis; hence, the vector  $Cv$  is equal to zero. Multiply  $Cv = 0$  by  $B$  to conclude  $BCv = 0$ . But,  $BC = I$  and therefore,  $v = 0$ .