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**Quiz for October 1, 2009**

Let  $W$  be a subspace of  $\mathbb{R}^n$  and let  $A$  be any  $m \times n$  matrix. Let  $V$  be the subset of  $\mathbb{R}^m$  defined by

$$V = \{y \in \mathbb{R}^m \mid y = Ax \text{ for some } x \text{ in } W\}.$$

Prove that  $V$  is a subspace of  $\mathbb{R}^m$ .

**ANSWER:**

- Zero is in  $V$ . The set  $W$  is a vector space; so  $0 \in W$  and  $A0 = 0$  is in  $V$ .
- The set  $V$  is closed under addition. Take  $v_1$  and  $v_2$  in  $V$ . The definition of  $V$  says that there are  $w_1$  and  $w_2$  in  $W$  with  $v_i = Aw_i$  for both  $i$ . The set  $W$  is a vector space; so  $W$  is closed under addition. Thus,  $w_1 + w_2 \in W$  and  $A(w_1 + w_2)$  is in  $V$ . On the other hand  $A(w_1 + w_2) = Aw_1 + Aw_2 = v_1 + v_2$ . We have shown that  $v_1 + v_2$  is in  $V$ .
- The set  $V$  is closed under scalar multiplication. Take  $v$  in  $V$  and  $c$  in  $\mathbb{R}$ . The definition of  $V$  says that there is a vector  $w$  in  $W$  with  $Aw = v$ . The set  $W$  is a vector space; so  $W$  is closed under scalar multiplication and therefore,  $cw$  is in  $W$ . It follows that  $A(cw)$  is in  $V$ . On the other hand,  $A(cw) = cAw = cv$ . We have shown that  $cv$  is in  $V$ .