Please PRINT your name ______________________________________

**No calculators, cell phones, computers, notes, etc.**

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

**Quiz 7, April 13, 2022**

Let $U$ and $V$ be finite dimensional subspaces of the vector space $W$.

Suppose that $z_1,\dots,z_c$ is a basis for $U\cap V$.

Suppose that $u_{c+1},\dots,u_a$ are elements in $U$ so that

$$z_1,\dots,z_c,u_{c+1},\dots,u_a$$

is a basis for $U$.

Suppose that $v_{c+1},\dots,v_b$ are elements in $V$ so that

$$z_1,\dots,z_c,v_{c+1},\dots,v_b$$

is a basis for $V$.

Prove that the elements

$$z_1,\dots,z_c,u_{c+1},\dots,u_a,v_{c+1},\dots,v_b \tag{1}$$

of $W$ are linearly independent.

**Answer:** Suppose

$$A_1,\dots,A_c,B_{c+1},\dots,B_a,C_{c+1},\dots,C_b$$

are numbers with

$$\sum_{i=1}^{c}A_iz_i+\sum_{j=c+1}^{a}B_ju_j+\sum_{k=c+1}^{b}C_kv_k=0.$$

Observe that

$$\sum_{i=1}^{c}A_iz_i+\sum_{j=c+1}^{a}B_ju_j=-\sum_{k=c+1}^{b}C_kv_k \tag{2}$$

is an element of $U\cap V$. The vectors $z_1,\dots,z_c$ are a basis for $U\cap V$; hence there are numbers $D_1,\dots,D_c$ with

$$\sum_{i=1}^{c}D_iz_i=-\sum_{k=c+1}^{b}C_kv_k.$$

However, the vectors $z_1,\dots,z_c,v_{c+1},\dots,v_b$ are a basis for $V$; thus, the vectors

$$z_1,\dots,z_c,v_{c+1},\dots,v_b$$

are linearly independent and $D_1 = \ldots, D_c = C_1 = \cdots = C_b = 0$. At this point (2) reads

$$\sum_{i=1}^{C} A_i z_i + \sum_{j=c+1}^{a} B_j u_j = 0.$$

However the vectors $z_1, \ldots, z_c, u_{c+1}, \ldots, u_a$ are a basis for $U$; thus,

$$z_1, \ldots, z_c, u_{c+1}, \ldots, u_a$$

are linearly independent and

$$A_1 = \cdots = A_c = B_{c+1} = \cdots = B_a = 0.$$

We have shown that the vectors (1) are linearly independent.