Please PRINT your name ______________________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 5, March 21, 2022

Let $\mathbb{V}$ be a vector space; let U and V be subspaces of $\mathbb{V}$; and let

$$W = \{w \in \mathbb{V} \mid w = u + v \text{ for some } u \in U \text{ and } v \in V\}.$$

Is W a vector space? Justify your answer completely. **Answer:** This W is a vector space.

The set W is closed under addition. Take w_1 and w_2 from W. Well, $w_1 = u_1 + v_1$ and $w_2 = u_2 + v_2$ for some $u_i \in U$ and $v_i \in V$.We see that

$$w_1 + w_2 = (u_1 + v_1) + (u_2 + v_2) = (u_1 + u_2) + (v_1 + v_2);$$

furthermore, $u_1 + u_2 \in U$ because U is a vector space and $v_1 + v_2$ is in V because V is a vector space. We conclude that $w_1 + w_2$ is equal to an element of U plus an element of V; and therefore, $w_1 + w_2$ is in W.

The set W is closed under scalar multiplication. Take $w_1 = u_1 + v_1 \in W$, as above, and $r \in \mathbb{R}$. We see that $rw_1 = ru_1 + rv_1$. The vector space U is closed under scalar multiplication; so, ru_1 is in U. Also, rv_1 is in V again because V is a vector space. Once again rw_1 has the correct form; that is rw_1 is equal to an element of U plus an element of V; therefore, rw_1 is in W.

The zero vector in $\mathbb{V}$ is equal to the zero vector of U plus the zero vector of V; and therefore, the zero vector is in W.