Let

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \,\middle|\, |x_1| + |x_2| = 0 \right\}.$$

Is *W* a vector space? Explain your answer thoroughly.

Answer. Yes, *W* is a vector space. Indeed *W* consists of one vector; namely the zero vector. Furthermore,

$$W = \left\{ \begin{bmatrix} 0\\0 \end{bmatrix} \right\}$$

is a vector space.

- The zero vector is in *W*.
- If one adds two elements of *W*, then both of these elements are the zero vector; hence the sum is also the zero vector which is in *W*.
- If one multiplies an element of W by a scalar, then the answer is the zero vector which is in W.

The zero vector all by itself is a vector space.