Quiz 1 Math 544, August 31, 2020

What is the solution set of the system of equations which corresponds to the augmented matrix

$$\left[\begin{array}{ccccc|c} 2 & 1 & 3 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 3 & 0 \end{array}\right] ?$$

ANSWER: We put the matrix in Reduced Row Echelon Form. Replace $R3$ with $(1/3)R3$ to obtain

$$\left[\begin{array}{ccccc|c} 2 & 1 & 3 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right].$$

Replace $R2$ with $R2-2R3$ to obtain

$$\left[\begin{array}{ccccc|c} 2 & 1 & 3 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right].$$

Replace $R1$ with $R1-3R2$

$$\left[\begin{array}{ccccc|c} 2 & 1 & 0 & -1 & 0 & -2 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right].$$

Replace $R1$ with $(1/2)R1$ to obtain

$$\left[\begin{array}{ccccc|c} 1 & 1/2 & 0 & -1/2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right].$$

The solution set is

$$\left\{ \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array}\right] \middle| \begin{array}{l} x_1 = -1 - 1/2x_2 + 1/2x_4, \\ x_3 = \quad 1 \qquad\qquad - x_4, \\ x_5 = \quad 0, \\ \text{and } x_2 \text{ and } x_4 \text{ are arbitrary} \end{array} \right\}.$$