

Please PRINT your name \_\_\_\_\_

**No calculators, cell phones, computers, notes, etc.**

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

### Quiz 1, January 19, 2022

Find the general solution of the following system of linear equations:

$$\begin{array}{rccccrcr} x_1 & + & x_2 & & & - & x_5 & = & 1 \\ & & & x_2 & + & 2x_3 & + & x_4 & + & 3x_5 & = & 1 \\ x_1 & & & & - & x_3 & + & x_4 & + & x_5 & = & 0. \end{array}$$

Also find **three** particular solutions of this system of equations. **Be sure to check** that all three of your particular solutions really satisfy the original system of linear equations.

**ANSWER: Solution:** We use the notation of augmented matrices:

$$\left[ \begin{array}{ccccc|c} 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 1 & 0 & -1 & 1 & 1 & 0 \end{array} \right].$$

Replace row 3 with row 3 minus row 1:

$$\left[ \begin{array}{ccccc|c} 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 0 & -1 & -1 & 1 & 2 & -1 \end{array} \right].$$

Replace row 1 with row 1 minus row 2 and  
replace row 3 with row 3 plus row 2:

$$\left[ \begin{array}{ccccc|c} 1 & 0 & -2 & -1 & -4 & 0 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 0 & 0 & 1 & 2 & 5 & 0 \end{array} \right].$$

Replace row 1 with row 1 plus 2 row 3 and  
replace row 2 with row 2 minus 2 row 3:

$$\left[ \begin{array}{ccccc|c} 1 & 0 & 0 & 3 & 6 & 0 \\ 0 & 1 & 0 & -3 & -7 & 1 \\ 0 & 0 & 1 & 2 & 5 & 0 \end{array} \right].$$

Our matrix is in reduced row echelon form. We read the answer. The general solution of the system of equations is

$$\begin{cases} x_1 = 0 - 3x_4 - 6x_5 \\ x_2 = 1 + 3x_4 + 7x_5 \\ x_3 = 0 - 2x_4 - 5x_5 \\ x_4 = x_4 \\ x_5 = x_5, \text{ where } x_4 \text{ and } x_5 \text{ are free to take any value.} \end{cases}$$

We consider the particular solutions when  $x_4 = x_5 = 0$ , when  $x_4 = 1$  and  $x_5 = 0$ , and when  $x_4 = 0$  and  $x_5 = 1$ . These solutions are

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} -3 \\ 4 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} -6 \\ 8 \\ -5 \\ 0 \\ 1 \end{bmatrix}.$$

We check the first particular solution:

$$\begin{array}{rcccccl} 0 & + & 1 & & & - & 0 & = & 1\checkmark \\ & & & 1 & +2(0) & + & 0 & +3(0) & = & 1\checkmark \\ 0 & & & - & (0) & + & 0 & + & 0 & = & 0\checkmark. \end{array}$$

We check the second particular solution:

$$\begin{array}{rcccccl} -3 & + & 4 & & & - & 0 & = & 1\checkmark \\ & & & 4 & +2(-2) & + & 1 & +3(0) & = & 1\checkmark \\ -3 & & & - & (-2) & + & 1 & + & 0 & = & 0\checkmark. \end{array}$$

We check the third particular solution:

$$\begin{array}{rcccccl} -6 & + & 8 & & & - & 1 & = & 1\checkmark \\ & & & 8 & +2(-5) & + & 0 & +3(1) & = & 1\checkmark \\ -6 & & & - & (-5) & + & 0 & + & 1 & = & 0\checkmark. \end{array}$$