

**Math 544, Exam 4, Fall 2005**

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you. There are 10 problems. Each problem is worth 5 points. The exam worth 50 points. **SHOW** your work. **CIRCLE** your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail.**

I will post the solutions on my website shortly after the exam is finished.

Recall that the matrix  $A$  is *symmetric* if  $A^T = A$  and the matrix  $A$  is *skew-symmetric* if  $A^T = -A$ .

1. Let  $A$  be a  $2 \times 2$  symmetric matrix with real number entries. Does  $A$  HAVE to have real eigenvalues? If yes, PROVE it. If no, then give a counter example.
2. Let  $A$  be a  $2 \times 2$  skew-symmetric matrix with real number entries. Does  $A$  HAVE to have real eigenvalues? If yes, PROVE it. If no, then give a counter example.
3. Let  $A$  be a  $3 \times 3$  skew-symmetric matrix with real number entries. Does  $A$  have to be singular? If yes, PROVE it. If no, then give a counter example.
4. Let  $A$  be a  $2 \times 2$  skew-symmetric matrix with real number entries. Does  $A$  have to be singular? If yes, PROVE it. If no, then give a counter example.
5. Find the inverse of

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & -1 & 1 \\ 1 & -1 & 1 & 0 \\ 1 & 0 & -1 & -1 \end{bmatrix}.$$

It might be worth your while to notice that the columns of  $A$  form an orthogonal set.

6. Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a linear transformation with  $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$  and  $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$ . Find a matrix  $M$  with  $T(v) = Mv$  for all vectors  $v$  in  $\mathbb{R}^2$ .

7. Let  $V$  be the set of  $2 \times 2$  singular matrices. Is  $V$  a vector space? Explain **thoroughly**.
8. Let  $V$  be the set of  $2 \times 2$  matrices with trace 0. Is  $V$  a vector space? Explain **thoroughly**. Recall that the *trace* of a square matrix is the sum of the elements on its main diagonal.
9. Find a matrix  $B$  with  $B^2 = A$  for  $A = \begin{bmatrix} 34 & 75 \\ -10 & -21 \end{bmatrix}$ . **Check your answer.**
10. Let  $A$  be a square matrix,  $v_1$  and  $v_2$  be non-zero vectors with  $Av_1 = \lambda_1 v_1$  and  $Av_2 = \lambda_2 v_2$ , where  $\lambda_1$  and  $\lambda_2$  are real numbers with  $\lambda_1 \neq \lambda_2$ . Prove that  $\{v_1, v_2\}$  is a linearly independent set of vectors.