Math 544, Exam 3, Spring, 2022

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

The exam is worth 50 points. Each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

No Calculators, Cell phones, computers, notes, etc.

(1) Define "linearly independent". Use complete sentences. Include everything that is necessary, but nothing more.

(2) Let A be an $n \times m$ matrix. Suppose that $v_1, \dots, v_a, w_1 \dots, w_b$ are vectors in $\mathbb{R}^n$ with $v_1, \dots, v_a$ linearly independent elements in the null space A, and $Aw_1, \cdots, Aw_b$ linearly independent elements in $\mathbb{R}^m$.
Prove that $v_1, \dots, v_a, w_1, \dots, w_b$ are linearly independent.

(3) Let V be the vector space of 4×4 skew-symmetric matrices. Give a basis for V. Recall that the square matrix A is *skew-symmetric* if $A^{\mathrm{T}} = -A$. Justify your answer.

(4) Let V be the vector space of polynomials $p(x)$ of degree at most four with the property that $\int_0^1 p(x)dx = 0$. Give a basis for V. Justify your answer.

(5) Let

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 & 3 & 3 \\ 1 & 3 & 4 & 2 & 3 & 3 \\ 1 & 3 & 4 & 1 & 3 & 1 \\ 1 & 3 & 4 & 2 & 3 & 1 \end{bmatrix}.$$

(a) Find a basis for the null space of A.

(b) Find a basis for the column space of A.

(c) Find a basis for the row space of A.

(d) Express each column of A in terms of your answer to (b).

(e) Express each row of A in terms of your answer to (c).