Math 544, Exam 3, Spring 2016

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. SHOW your work. **No Calculators or Cell phones.** Write your answers as legibly as you can. Make your work be coherent and clear. Write in complete sentences. I will post the solutions on my website shortly after the exam is finished.

1. Let $A = \begin{bmatrix} 1 & 4 & -2 & 1 & 5 & 5 & 5 \\ 1 & 4 & -2 & 2 & 8 & 9 & 7 \\ 2 & 8 & -4 & 3 & 13 & 14 & 0 \\ 3 & 12 & -6 & 5 & 21 & 23 & 7 \end{bmatrix}$.
   Find a basis for the null space of A. Find a basis for the column space of A. Find a basis for the row space of A. Express each column of A in terms of your basis for the column space. Express each row of A in terms of your basis for the row space. **Check your answer.**

2. Define basis. **Use complete sentences. Say everything that has to be said and nothing more.**

3. Define dimension. **Use complete sentences. Say everything that has to be said and nothing more.**

4. Let A be an $n \times m$ matrix and V be a subspace of $\mathbb{R}^m$. Define N and C to be the following vector spaces
   $$N = \{v \in V | Av = 0\} \quad \text{and} \quad C = \{Av \mid v \in V\}.$$
   Let $u_1, \dots, u_p$ be vectors in V with $Au_1, \dots, Au_p$ a basis for C and let $v_1, \dots, v_q$ be a basis for N. Prove that the vectors $u_1, \dots, u_p, v_1, \dots, v_q$ span V. (You will have to write a proof from scratch. We have not proven this particular statement before.)

5. Let $U_1 \subseteq U_2 \subseteq U_3 \subseteq \mathbb{R}^4$ be vector spaces. Suppose v_1, v_2, v_3, v_4 is a basis for $\mathbb{R}^4$, $v_1, v_2, v_3 \in U_3$, $v_4 \notin U_3$; $v_1, v_2 \in U_2$, $v_3 \notin U_2$; and $v_1 \in U_1$, $v_2 \notin U_1$. Tell as much as you can about $\dim U_1$, $\dim U_2$, and $\dim U_3$. Prove any statements that you make.