Math 544, Exam 2, Fall, 2020

Write everything on the blank paper that you brought. There should be nothing on your desk except this exam, the blank paper that you brought, and a pen or pencil. When you are finished, send a picture of your solutions to

kustin@math.sc.edu

You should KEEP this piece of paper. If possible: put the problems in order before you take your picture. (Use as much paper as necessary).

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

(1) Define "column space". Use complete sentences. Include everything that is necessary, but nothing more.

(2) Let A and B be $n \times n$ matrices. Is the column space of AB always contained in the column space of A? If yes, prove the assertion. If no, give an example.

(3) Let A and B be $n \times n$ matrices. Is the column space of A always contained in the column space of AB? If yes, prove the assertion. If no, give an example.

(4) Let A and B be $n \times n$ matrices, with B non-singular. Is the column space of A always contained in the column space of AB? If yes, prove the assertion. If no, give an example.

(5) Let A be an $n \times n$ matrix. Suppose that $v_1, \dots, v_s$ are vectors in $\mathbb{R}^n$ with $Av_1, \dots, Av_s$ linearly independent, and that $w_1, \dots, w_t$ are linearly independent vectors in the null space of A. Prove that $v_1, \dots, v_s, w_1, \dots, w_t$ are linearly independent vectors.