## Math 544, Exam 2, Fall 2009

Write your answers as legibly as you can.
There are 7 problems. Problem 7 is worth 22 points. Each of the other problems is worth 13 points. The exam is worth a total of 100 points. SHOW your work. CIRCLE your answer. CHECK your answer whenever possible. No Calculators.

I will post the solutions on my website shortly after the class is finished.

1. Let $U \subseteq V$ be subspaces of $\mathbb{R}^{n}$ with $\operatorname{dim} U=\operatorname{dim} V$. Do $U$ and $V$ HAVE to be equal? If yes, prove your answer. If no, give an example.
2. Let $A$ and $B$ be $n \times n$ matrices. Does the null space of $A B$ HAVE to be a subset of the null space of $A$ ? If yes, prove your answer. If no, give an example.
3. Define "null space". Use complete sentences. Include everything that is necessary, but nothing more.
4. Define "dimension". Use complete sentences. Include everything that is necessary, but nothing more.
5. Let

$$
V=\left\{\left.\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right] \in \mathbb{R}^{3} \right\rvert\, 2 x_{1}+3 x_{3}-4 x_{3}=5\right\}
$$

Is $V$ a vector space? Explain thoroughly.
6. Let $a=\left[\begin{array}{l}a_{1} \\ a_{2} \\ a_{3}\end{array}\right]$ and $b=\left[\begin{array}{l}b_{1} \\ b_{2} \\ b_{3}\end{array}\right]$ be fixed elements of $\mathbb{R}^{3}$, and let

$$
V=\left\{\left.x=\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right] \in \mathbb{R}^{3} \right\rvert\, a^{\mathrm{T}} x=0 \text { and } b^{\mathrm{T}} x=0\right\} .
$$

Is $V$ a vector space? Explain thoroughly.
7. Let $A$ be the matrix

$$
A=\left[\begin{array}{ccccc}
1 & 3 & 4 & 2 & 4 \\
1 & 3 & 4 & 3 & 6 \\
2 & 6 & 8 & 5 & 10
\end{array}\right]
$$

(a) Find a basis for the null space of $A$.
(b) Find a basis for the column space of $A$.
(c) Find a basis for the row space of $A$.
(d) Write each column of $A$ as a linear combination of your answer to (b).
(e) Write each row of $A$ as a linear combination of your answer to (c).

