Math 544, Exam 2, Spring 2016

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **7** problems on **two sides**. SHOW your work. **No Calculators or Cell phones.** Write your answers as legibly as you can. Make your work be coherent and clear. Write in complete sentences. I will post the solutions on my website shortly after the exam is finished.

1. (8 points) Let v_1 and v_2 be fixed vectors in $\mathbb{R}^n$, for some n, and let

$$V = \{v \in \mathbb{R}^n \mid v_1^{\mathrm{T}} v = 0 \text{ and } v_2^{\mathrm{T}} v = 0\}.$$

Is V a vector space? If yes, prove the statement. If no, show an example in which one of the rules of vector spaces is violated.

2. (7 points) Let

$$V = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, ab = 0 \right\}.$$

Is V a vector space? If yes, prove the statement. If no, show that one of the rules of vector spaces is violated.

3. (7 points) Let A and B be 2×2 matrices with $A^2 = B^2$. Does B have to equal A or $-A$? If yes, prove the statement. If no, show an example.

4. (7 points) Consider the vectors

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \text{and} \quad v_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

Are the vectors v_1, v_2, v_3 linearly independent? Demonstrate that your answer is correct. (It might be useful to notice that $v_i^{\mathrm{T}} v_j = 0$ whenever $i \neq j$.)

5. (7 points) Let A and B be $n \times n$ matrices with $AB = I$. Does BA have to equal I? If yes, prove the statement. If no, show an example.

6. (7 points) Let A be an $m \times n$ matrix and B be an $n \times m$ matrix with $AB = I$. Does BA have to equal I? If yes, prove the statement. If no, show an example.

Please turn over.

7. Consider the system of equations $Ax = b$ where $A = \begin{bmatrix} a & -3a+3 \\ 1 & a-1 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and $b = \begin{bmatrix} -9 \\ 3 \end{bmatrix}$.

   (a) For which values of a does the system of equations have no solution?
   (b) For which values of a does the system of equations have exactly one solution?
   (c) For which values of a does the system of equations have more than one solution?