Math 544, Exam 1, Summer 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **7** problems on **TWO SIDES**. SHOW your work. **No Calculators or Cell phones.** Write your answers as legibly as you can. Make your work be coherent and clear. Write in complete sentences. I will post the solutions on my website shortly after the exam is finished.

1. (8 points) Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations. $\boxed{CIRCLE}$ your answer.

$$A = \begin{bmatrix} 1 & 2 & 1 & 7 & 1 \\ 1 & 2 & 2 & 10 & 1 \\ 1 & 2 & 1 & 7 & 2 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad b = \begin{bmatrix} 15 \\ 20 \\ 21 \end{bmatrix}.$$

2. (7 points) **Define** "linearly independent". Use complete sentences. Include everything that is necessary, but nothing more.

3. (7 points) Let A and B be 2×2 matrices with A not equal to the zero matrix and $BA = A^2$. Does B have to equal A? If yes, prove your answer. If no, give a counterexample.

4. (7 points) Let A and B be 2×2 symmetric matrices. Does the product AB have to be a symmetric matrix? If yes, prove your answer. If no, give a counterexample.

5. (7 points) Let v_1, v_2, v_3 be linearly independent vectors in $\mathbb{R}^3$. Define v_1^*, v_2^*, v_3^* to be the vectors

$$v_1^* = \begin{bmatrix} v_1 \\ 1 \end{bmatrix}, \quad v_2^* = \begin{bmatrix} v_2 \\ 1 \end{bmatrix}, \quad v_3^* = \begin{bmatrix} v_3 \\ 1 \end{bmatrix}$$

in $\mathbb{R}^4$. Do the vectors v_1^*, v_2^*, v_3^* have to be linearly independent? If yes, prove your answer. If no, give a counterexample.

There are more problems on the other side.

6. (7 points) Consider the system of linear equations.

$$\begin{aligned} x_1 + (a-1)x_2 &= 4 \\ ax_1 + \quad\quad 6x_2 &= 12. \end{aligned}$$

(a) Which values for a cause the system to have no solution?
(b) Which values for a cause the system to have exactly one solution?
(c) Which values for a cause the system to have an infinite number of solutions?

Explain thoroughly.

7. (7 points) Are the vectors

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

linearly independent? **Explain thoroughly.**