Math 544, Exam 1, Spring 2011

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **9** problems on **TWO SIDES**. SHOW your work. **No Calculators or Cell phones.** Write your answers as legibly as you can. Make your work be coherent and clear. Write in complete sentences. I will post the solutions on my website shortly after the exam is finished.

1. (10 points) Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations. $\boxed{CIRCLE}$ your answer.

$$A = \begin{bmatrix} 1 & 4 & 5 & 1 & 8 \\ 1 & 4 & 5 & 2 & 10 \\ 3 & 12 & 15 & 4 & 26 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix}.$$

2. (5 points) **Define** "linearly independent". Use complete sentences. Include everything that is necessary, but nothing more.

3. (5 points) **Define** "non-singular". Use complete sentences. Include everything that is necessary, but nothing more.

4. (5 points) **State** the result about the linear dependence or linear independence p vectors in $\mathbb{R}^m$. Include everything that is necessary, but nothing more.

5. (5 points) Let A be a non-singular $n \times n$ matrix and let b be an element of $\mathbb{R}^n$. **Prove** that $Ax = b$ has at least one solution. (I want a **complete proof**. The answer "We did this in class" is not acceptable.)

6. (5 points) Let A, B, and C be 2×2 matrices with A not equal to the zero matrix and $BA = CA$. Does B have to equal C? If yes, prove your answer. If no, give a counterexample.

7. (5 points) Let A and B be 2×2 matrices. Does $(A + B)(A - B)$ have to equal $A^2 - B^2$? If yes, prove your answer. If no, give a counterexample.

There are more problems on the other side.

8. (5 points) Let A and B be $n \times n$ matrices. How is $(AB)^{\mathrm{T}}$ related to the product of A^{T} and B^{T} ? Prove that your answer is correct.

9. (5 points) Let v_1, v_2, v_3, v_4 be vectors in $\mathbb{R}^4$ with v_1, v_2, v_3 linearly dependent. Do v_1, v_2, v_3, v_4 have to be linearly dependent? If yes, prove your answer. If no, give a counterexample.